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## Adaptive and Optimal Control of Stochastic Dynamical Systems

Tyrone Duncan  
UNIVERSITY OF KANSAS CENTER FOR RESEARCH INC.

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09/14/2015  
Final Report

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# Final Report for AFOSR Grant FA9550-12-1-0384 Adaptive and Optimal Control of Stochastic Dynamical Systems

Tyrone E. Duncan and Bozenna Pasik-Duncan \*

## Abstract

A number of explicit results have been obtained for problems of stochastic control and stochastic differential games. Stochastic linear-quadratic, continuous time, stochastic control problems are solved for systems with noise processes that are arbitrary fractional Brownian motions or more generally square integrable continuous processes. Linear-quadratic control problems for stochastic partial differential equations with fractional Brownian motions having the Hurst parameter in the interval  $(1/2,1)$ , which includes the long range dependent processes, are solved. A direct method for solving stochastic control problems is given that does not require the solution of Hamilton-Jacobi-Bellman equations or the solution of backward stochastic differential equations. Both of these latter methods present serious difficulties for solutions. A stochastic control problem is explicitly solved for a system that evolves in the two-sphere which is useful for applications. Linear exponential quadratic Gaussian control problems for both continuous and discrete time systems are solved in a simple direct way. Linear exponential-quadratic control problems for stochastic partial differential equations are explicitly solved. Discrete time linear quadratic control problems for systems with arbitrary correlated noise are explicitly solved. Linear-quadratic stochastic differential games are explicitly solved for systems with arbitrary noise.

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Stochastic differential games for stochastic partial differential equations with fractional Brownian motions are explicitly solved. Ergodic control problems for linear exponential quadratic control for stochastic partial differential equations with fractional Brownian motions are explicitly solved. Stochastic differential games that evolve in spheres are explicitly solved. Various aspects of stochastic adaptive control are described. Some linear quadratic control problems with state dependent noise are explicitly solved. Optimal controls for partially observed linear exponential quadratic Gaussian control are obtained given the information filter.

## 1 Introduction

This report provides a description of the variety of results that were obtained during the three year AFOSR grant. A major development was an approach to problems of stochastic control or stochastic differential games that does not require finding solutions to nonlinear partial differential equations or solving backward stochastic differential equations. This direct approach allowed for the consideration of stochastic models that have general noise processes and it allowed for explicit results. This direct approach allowed for the two researchers supported on this grant to consider both continuous and discrete time stochastic control systems. Furthermore it allowed to consider the control of stochastic partial differential equations that are driven by fractional Brownian motions. The stochastic differential games that were solved provided explicit control strategies that formed Nash equilibria.

## 2 Description of Main Results

Optimal control problems for linear stochastic equations with noise that is square integrable with continuous sample paths and quadratic cost functionals are solved to obtain explicit optimal controls [1, 3, 9, 19]. These results are significant generalizations of the well known linear-quadratic Gaussian control problems. The approach to obtain explicit optimal stochastic controls [10] does not require solving Hamilton-Jacobi-Bellman partial differential equations or solving backward stochastic differential equations from a stochastic maximum principle. The control of linear stochastic partial differential equations with quadratic cost, fractional Brownian motions and infi-

nite time horizons is considered where explicit optimal controls are obtained [2, 17]. Optimal controls for discrete time linear systems with quadratic costs and arbitrary correlated noise are given explicitly in [6]. A variety of stochastic differential games are considered for both finite and infinite dimensional linear stochastic systems. Optimal control strategies for linear-quadratic stochastic differential games with general noise processes are given explicitly [11]. stochastic differential games described by stochastic partial differential equations with fractional Brownian motions are solved in [12]. Linear-exponential-quadratic control for Brownian motion noise are solved in both finite and infinite dimensional spaces [5, 7, 8, 12, 13]. Ergodic control problems for linear stochastic partial differential equations with fractional Brownian motions are solved in [18]. Explicit control strategies for stochastic differential games that evolve in the unit sphere in Euclidean three space are given in [4, 14, 16]. A description of stochastic adaptive control is given in [15]. Some results for linear-quadratic control with state dependent noise is given in [21]. Optimal controls for partially observed linear-exponential-Gaussian control given the information filter for the estimation is solved directly in [20].

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**Abstract**

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**Changes in research objectives (if any):**

The research has been expanded from stochastic control to include stochastic differential games. The problems have included general system noise models and nonlinear stochastic systems.

**Change in AFOSR Program Manager, if any:**

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